Quiz #1; Tuesday, date: 01/23/2018
MATH 53 Multivariable Calculus with Stankova
Section #114; time: 2 – 3:30 pm
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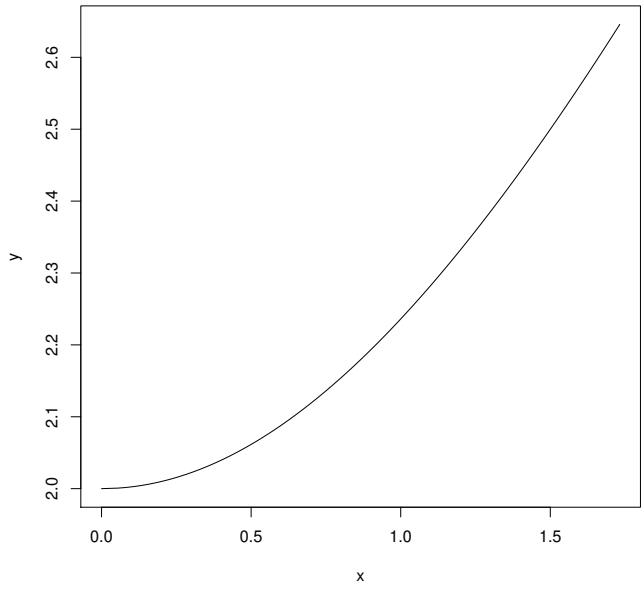
1. Consider the parametric equation for a curve:

$$x = \sqrt{t-1}, \qquad y = \sqrt{t+3}.$$

Eliminate the parameter to find a Cartesian equation of the curve. Sketch the curve and indicate with an arrow the direction in which the curve is traced as the parameter increases.

Solution. There's not much that can be done with the equations in this form, so try squaring both sides: $x^2 = t - 1$ and $y^2 = t + 3$, so $y^2 - x^2 = 4$. The points therefore lie on a hyperbola, but the restrictions of the parametric equations also mean that x and y must both be non-negative.



Both x and y are increasing functions of t, but x is defined only when $t \geq 1$. The particle starts at $(0, 2)$ when $t = 1$ and moves upwards (and rightwards) as $t \to \infty$.

2. *True / False?* A Cartesian equation $f(x, y) = 0$ of a curve in the plane can always be re-written to define the curve by some function: $y = g(x)$, or by some function: $x = h(y)$.

   Solution. **False**; consider the circle $x^2 + y^2 = 1$. It cannot simply be written as a function as it does not pass the vertical line test.

3. *True / False?* The polar curve

$$r = 2\cos\theta, \qquad 0 \leq \theta \leq 6\pi$$

   is a circle centered at $(1, 0)$, traversed 6 times.

   Solution. **True**; by plugging in some value, one should realize that from 0 to π the circle is traversed once, then proceed to repeat the same path. Since the restriction on θ is from 0 to 6π, the circle is traversed 6 times.