Quiz #2; Tuesday, date: 01/23/2018
MATH 53 Multivariable Calculus with Stankova
Section #117; time: 5 – 6:30 pm
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1. Find the slope of the tangent line to the given polar curve at the point specified by the value of θ:

$$r = 1 + \sqrt{2}\cos\theta, \quad \theta = \pi/4.$$

Solution. The slope of the tangent is given by

$$\begin{aligned}\frac{dy}{dx} &= \frac{\frac{dr}{d\theta}\sin\theta + r\cos\theta}{\frac{dr}{d\theta}\cos\theta - r\sin\theta} \\ &= \frac{-\sqrt{2}\sin^2\theta + \cos\theta + \sqrt{2}\cos^2\theta}{-\sqrt{2}\sin\theta\cos\theta - \sin\theta - \sqrt{2}\sin\theta\cos\theta} \\ &= \frac{\sqrt{2}\cos 2\theta + \cos\theta}{-\sqrt{2}\sin 2\theta - \sin\theta}.\end{aligned}$$

Plugging in $\pi/4$ gives

$$\frac{\sqrt{2}\cdot 0 + 1/\sqrt{2}}{-\sqrt{2} - 1/\sqrt{2}} = -\frac{1}{3}.$$

2. *True / False?* It is possible to compute the arc length of a polar curve in form of $r = f(\theta)$ using the arc length formula for parametric curves.

Solution. True. The polar curve can be regarded as the parametric equation

$$x = f(\theta)\cos\theta, \quad y = f(\theta)\sin\theta.$$

It is just more cumbersome to compute with this instead of the polar curve arc length integral.

3. *True / False?* The sum of two unit vectors is always a unit vector.

Solution. False. The length of the sum of two unit vectors is not necessarily 1. In fact, it can be as large as 2 (try adding $\mathbf{i}$ to itself) and as small as 0 (try adding $\mathbf{i}$ to $-\mathbf{i}$), or something in between (try adding $\mathbf{i}$ to $\mathbf{j}$).