Quiz #3; Tuesday, date: 02/06/2018
MATH 53 Multivariable Calculus with Stankova
Section #114; time: 2 – 3:30 pm
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1. Find the scalar and vector projections of $\mathbf{b}$ onto $\mathbf{a}$.

$$\mathbf{a} = \langle -6, 3, -2 \rangle, \qquad \mathbf{b} = \langle 4, -1, 4 \rangle.$$

Solution. We apply the formula for scalar projection first.

$$\begin{aligned} \frac{\mathbf{a} \cdot \mathbf{b}}{|\mathbf{a}|} &= \frac{\langle -6, 3, -2 \rangle \cdot \langle 4, -1, 4 \rangle}{|\langle -6, 3, -2 \rangle|} \\ &= \frac{-35}{\sqrt{49}} \\ &= -5. \end{aligned}$$

We multiple this to a unit vector in the direction of $\mathbf{a}$ to find the vector projection next.

$$\begin{aligned} -5\frac{\mathbf{a}}{|\mathbf{a}|} &= -5\frac{\langle -6, 3, -2 \rangle}{\sqrt{49}} \\ &= \frac{5}{7}\langle 6, -3, 2 \rangle. \end{aligned}$$

2. *True / False?* For any vector $\mathbf{a}$ and $\mathbf{b}$, we have

$$(\mathbf{a} + \mathbf{b}) \times (\mathbf{a} - \mathbf{b}) = \mathbf{a} \times \mathbf{a} - \mathbf{b} \times \mathbf{b} = \mathbf{0} - \mathbf{0} = \mathbf{0}.$$

Solution. False. We start by simplifying the left hand side.

$$\begin{aligned} (\mathbf{a} + \mathbf{b}) \times (\mathbf{a} - \mathbf{b}) &= \mathbf{a} \times \mathbf{a} - \mathbf{a} \times \mathbf{b} + \mathbf{b} \times \mathbf{a} - \mathbf{b} \times \mathbf{b} \\ &= \mathbf{0} - \mathbf{a} \times \mathbf{b} - \mathbf{a} \times \mathbf{b} - \mathbf{0} \\ &= -2(\mathbf{a} \times \mathbf{b}), \end{aligned}$$

which is only 0 if $\mathbf{a}$ and $\mathbf{b}$ are scalar multiples.

3. *True / False?* In the three dimensional space, two lines must be in one of three cases:

   (a) they are parallel
   (b) they intersect
   (c) they do not lie on the same plane.

Solution. True. Recall the definition of *skew lines* is two lines not on the same plane. Two lines on the same plane either intersect or are parallel.