Quiz #4; Tuesday, date: 02/13/2018
MATH 53 Multivariable Calculus with Stankova
Section #114; time: 2 – 3:30 pm
GSI name: Kenneth Hung
Student name: SOLUTIONS

1. Reduce the equation to one of the standard forms, classify the surface, and sketch it.

$$x^2 - y^2 - z^2 + 2x - 6z - 8 = 0.$$

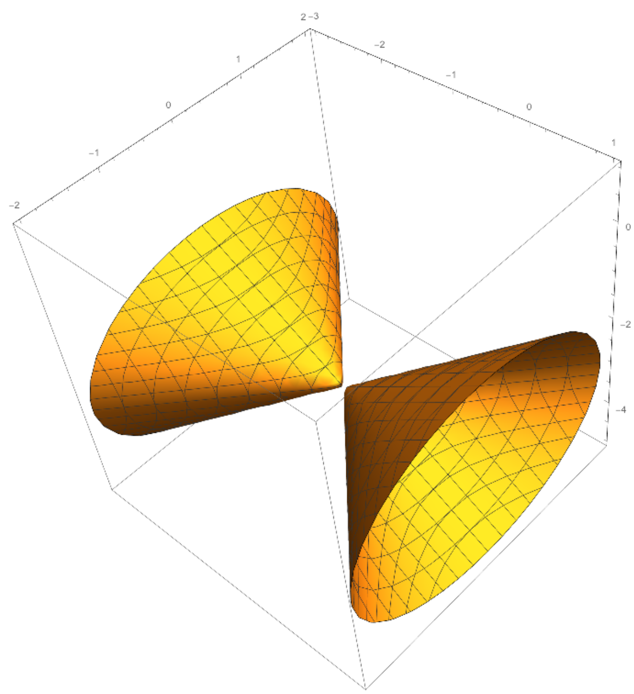
Solution. We start by completing the squares:

$$\begin{aligned} x^2 - y^2 - z^2 + 2x - 6z - 8 &= 0 \\ (x^2 + 2x) - y^2 - (z^2 + 6z) - 8 &= 0 \\ (x+1)^2 - y^2 - (z+3)^2 &= 0. \end{aligned}$$

Now we can this into a standard form:

$$(x+1)^2 = y^2 + (z+3)^2,$$

which is the equation of a cone oriented alnog the x-axis (i.e. elliptic cross sections at $x = k$), centered at $(-1, 0, -3)$. Sketch is as follows:



2. *True / False?* Consider a space curve given by the vector equation $\mathbf{r}(t)$. If all of its projections onto xy-plane, yz-plane and xz-plane are smooth, then the curve itself must be smooth.

   Solution. **True.** Suppose there is such a curve that is not smooth. Then its derivative $\mathbf{r}'(t)$ must not be defined at some point or is the zero vector. One of its projections onto xy-plane, yz-plane or xz-plane must then have undefined derivative or zero derivative at the same t-value, meaning that this projection will not be smooth.

3. *True / False?* One of the ways to visualize a space curve is to show it on a surface.

   Solution. **True.** For example to draw the helix, one can draw a cylinder first and show the helix as a curve of the surface of the cylinder for better visualization.