Quiz #4; Tuesday, date: 02/13/2018
MATH 53 Multivariable Calculus with Stankova
Section #117; time: 5 – 6:30 pm
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1. At what point do the curves $\mathbf{r}_1(t) = \langle t, 1-t, 8+t^2\rangle$ and $\mathbf{r}_2(s) = \langle 4-s, s-3, s^2\rangle$ intersect? Find their angle of intersection and leave the answer using inverse trigonometric function if necessary.

   Solution. At the intersection, the s-value and the t-value must satisfy

   $$\begin{cases} t &= 4-s \\ 1-t &= s-3 \\ 8+t^2 &= s^2 \end{cases}$$

   Substituting $t = 4-s$ into the last equation gives

   $$\begin{aligned} 8+(4-s)^2 &= s^2 \\ 24-8s+s^2 &= s^2 \\ s &= 3, \end{aligned}$$

   and so $t = 1$. The intersection coordinates can be obtained by plugging in either of these, and is $(1, 0, 9)$.

   The derivatives of $\mathbf{r}_1(t)$ and $\mathbf{r}_2(s)$ are

   $$\langle 1, -1, 2t\rangle \qquad \text{and} \qquad \langle -1, 1, 2s\rangle$$

   respectively, and the tangent vectors at the intersection are

   $$\langle 1, -1, 2\rangle \qquad \text{and} \qquad \langle -1, 1, 6\rangle$$

   respectively. So the angle between them is

   $$\cos^{-1}\frac{\langle 1, -1, 2\rangle \cdot \langle -1, 1, 6\rangle}{|\langle 1, -1, 2\rangle|\,|\langle -1, 1, 6\rangle|} = \cos^{-1}\frac{10}{\sqrt{6}\sqrt{38}} = \cos^{-1}\frac{5}{\sqrt{57}}.$$

2. *True / False?* The equation

   $$\frac{x^2}{a} + \frac{y^2}{b} + \frac{z^2}{c} = 1$$

   can give rise to different quadric surfaces, depending on the number of negative signs among a, b and c. Since there can be 0, 1, 2 and 3, this can be four different types of quadric surface.

   Solution. **False.** a, b and c cannot all be negative, as the left hand side would then always be negative when the right hand side is the positive number 1. There are only three possible types of quadric surface in this case: ellipsoid, hyperboloid of one sheet and hyperboloid of two sheets.

3. *True / False?* For any space curve with vector equation $\mathbf{r}(t)$, since $\mathbf{r}(t) \times \mathbf{r}(t) = \mathbf{0}$, we have

$$\mathbf{0} = \frac{d}{dt}[\mathbf{r}(t) \times \mathbf{r}(t)] = \mathbf{r}'(t) \times \mathbf{r}(t) + \mathbf{r}'(t) \times \mathbf{r}(t) = 2\mathbf{r}'(t) \times \mathbf{r}(t),$$

and so $\mathbf{r}'(t)$ must either be parallel to or in the opposite direction of $\mathbf{r}(t)$.

Solution. **False.** The product rule for cross products at the second equal sign would have given

$$\mathbf{r}'(t) \times \mathbf{r}(t) + \mathbf{r}(t) \times \mathbf{r}'(t) = \mathbf{0},$$

which does not prove anything. In fact for most curves, such as $\mathbf{r}(t) = \langle t, t^2, t^3 \rangle$, we would not have $\mathbf{r}(t)$ and $\mathbf{r}'(t)$ being scalar multiples of each other.