Quiz #5; Tuesday, date: 02/20/2018
MATH 53 Multivariable Calculus with Stankova
Section #114; time: 2 – 3:30 pm
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1. At what point does the curve have maximum curvature? What happens to the curvature as $x \to \pi/2$?

$$y = \ln(\sec x), \qquad 0 \le x < \frac{\pi}{2}.$$

Solution. To apply the formula for curvature, we need the first two derivatives, which are

$$y' = \frac{\sec x \tan x}{\sec x} = \tan x,$$
$$y'' = \sec^2 x.$$

Now by Formula 11 on pg. 865, we have

$$\kappa(x) = \frac{|\sec^2 x|}{\left(1 + \tan^2 x\right)^{3/2}} = \frac{\sec^2 x}{\sec^3 x} = \cos x.$$

The curve has maximum curvature at $x = 0$, and the curvature approaches 0 as $x \to \pi/2$.

2. *True / False?* If a curve is parametrized by its arc length, there is no tangential component of acceleration and the normal component of acceleration is the curvature.

Solution. **True.** Since the curve is parametrized by its arc length, we have $v(t) = 1$. By Formula 7 on pg. 874, the tangential component is v', which is zero, and the normal component is $\kappa v^2 = \kappa$, which is just the curvature.

3. *True / False?* The level surfaces of $f(x, y, z) = x^2 + y^2 - z$ are elliptic paraboloids, that can be obtained from each other by shifting in the z-direction.

Solution. **True.** The level surfaces are $x^2 + y^2 - z = k$ for some k, or equivalently,

$$z + k = x^2 + y^2,$$

which can be obtained by shifting the elliptic paraboloid $z = x^2 + y^2$ by k units in the negative z-direction.