Quiz #5; Tuesday, date: 02/20/2018
MATH 53 Multivariable Calculus with Stankova
Section #117; time: 5 – 6:30 pm
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1. Find the tangential and normal components of the acceleration vector.

$$\mathbf{r}(t) = t\mathbf{i} + 4e^{t/2}\mathbf{j} + 2e^t\mathbf{k}$$

Solution. We want to apply the formula for the two components. We start by computing the derivatives.

$$\mathbf{r}'(t) = \mathbf{i} + 2e^{t/2}\mathbf{j} + 2e^t\mathbf{k},$$
$$\mathbf{r}''(t) = e^{t/2}\mathbf{j} + 2e^t\mathbf{k}.$$

By Formula 9 and 10 on pg. 875,

$$a_T = \frac{\mathbf{r}'(t)\cdot\mathbf{r}''(t)}{|\mathbf{r}'(t)|} = \frac{2e^t + 4e^{2t}}{\sqrt{1+4e^t+4e^{2t}}} = 2e^t.$$
$$a_N = \frac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|} = \frac{|2e^{3t/2}\mathbf{i} - 2e^t\mathbf{j} + e^{t/2}|}{1+2e^t} = e^{t/2}.$$

2. *True / False?* Suppose the curve $\mathbf{r}(t)$ goes through the origin. A new curve formed by shrinking the curve $\mathbf{r}(t)$ towards the origin by a factor of 2. (In other words, a point $\mathbf{v}$ is shrunk to $\mathbf{v}/2$.) The curvature at the origin is doubled.

   Solution. **True.** Suppose the new curve is parametrized by $\mathbf{q}(t) = \mathbf{r}(t)/2$. The curvature of $\mathbf{q}$ is given by

$$\kappa_{\mathbf{q}}(t) = \frac{|\mathbf{q}'(t)\times\mathbf{q}''(t)|}{|\mathbf{q}'(t)|^3} = 2\frac{|\mathbf{r}'(t)\times\mathbf{r}''(t)|}{|\mathbf{r}'(t)|^3} = 2\kappa_{\mathbf{r}}(t).$$

   In particular, the curvature at the origin is doubled.

3. *True / False?* For a smooth space curve $\mathbf{r}(t)$ that is on the x, y-plane, the binormal vector (when defined) must either be $\mathbf{k}$ for all t or $-\mathbf{k}$ for all t, depending on which way the curve is traversed.

   Solution. **False.** While the binormal vector must either be $\mathbf{k}$, $-\mathbf{k}$, the sign may switch whenever the curvature is zero. In other words, for a curve with both counterclockwise and clockwise part, the binormal vector can be $\mathbf{k}$ some time and $-\mathbf{k}$ in some other time.