Quiz #6; Tuesday, date: 02/27/2018
MATH 53 Multivariable Calculus with Stankova
Section #114; time: 2 – 3:30 pm
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1. Find the limit, if it exists, or show that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{xy^3}{x^2+y^6}$$

Solution. On the x-axis, $f(x,0)=0$ for $x \neq 0$, so $f(x,y)\to 0$ as $(x,y)\to(0,0)$ along the x-axis. Approaching $(0,0)$ along the curve $x=y^3$ gives $f(y^3,y)=y^6/2y^6=\frac{1}{2}$ for $y\neq 0$, so along this path $f(x,y)\to\frac{1}{2}$ as $(x,y)\to(0,0)$. Thus the limit does not exist.

2. *True / False?* If f is a function whose domain contains points arbitrarily close to $(2,3)$, then

$$\lim_{(x,y)\to(2,3)} f(x,y)=(2,3).$$

Solution. **False.** From the definition of continuity, the above is only true if the function is continuous at $(2,3)$. In general functions may not be continuous at $(2,3)$, e.g.

$$f(x,y)=\begin{cases}1 & \text{if } (x,y)=(2,3)\\ 0 & \text{otherwise}\end{cases}$$

3. *True / False?* Consider two functions f and g that are both defined on the domain of f. Suppose the domain of f, D_f is contained in the domain of g, D_g (i.e. D_f is a subset of D_g) and $f(x)=g(x)$ for any points x in D_f. If the origin is in D_f and f is continuous at the origin, then g is also continuous at the origin.

Solution. **False.** Even the function is the same, expanding the domain can make a continuous function no longer continuous. Consider the example

$$f(x,y)=0 \text{ with domain } \{(x,y): x=0\}$$
$$g(x,y)=\begin{cases}0 & \text{if } x=0\\ 1 & \text{if } x=1\end{cases} \text{ with domain } \mathbb{R}^2$$

The two functions are equal in the domain of f and the domain of g contains the domain of g. The values of the function g in the larger domain may not conform to the requirements of continuity. Hence it is possible for f to be continuous at the origin with g not continuous at the origin.