Quiz #6; Tuesday, date: 02/27/2018
MATH 53 Multivariable Calculus with Stankova
Section #117; time: 5 – 6:30 pm
GSI name: Kenneth Hung
Student name: SOLUTIONS

1. Find the limit, if it exists, or show that the limit does not exist.

$$\lim_{(x,y)\to(0,0)} \frac{4y^2\cos^2 x}{x^2+y^2}$$

   Solution. First approach $(0,0)$ along the x-axis. Then $f(x,0) = 0/x^2 = 0$ for $x \neq 0$, so $f(x,y) \to 0$. Next approach $(0,0)$ along the y-axis. For $y \neq 0$, $f(0,y) = 4y^2/y^2 = 4$, so $f(x,y) \to 4$. Since f has two different limits along two different lines, the limit does not exist.

2. *True / False?* The function $f(x,y) = \sqrt{x-y+1}$ is not continuous at the point $(0,1)$.

   Solution. **False.** For any points (x,y) within distance $\epsilon^2/\sqrt{2}$ from $(0,1)$ in the domain, $\{(x,y) : x-y+1 \geq 0\}$, we have $0 \leq x-y+1 \leq \epsilon^2$, which means that $0 \leq x-y+1 \leq \epsilon$ and

$$|f(x,y) - f(0,1)| \leq \epsilon.$$

   Note that the definition of continuity at a point (a,b) disregards the points near (a,b) but outside of the domain.

3. *True / False?* To show that the limit at a point (a,b) exists, it suffices to find two paths to the point (a,b) where the limits of $f(a,b)$ agree.

   Solution. **False.** Checking all paths is not sufficient (refer to T/F Question 2 from Homework 15), not to say checking just two paths. Checking two paths is only sufficient for showing the limit does not exists.