Quiz #7; Tuesday, date: 03/06/2018
MATH 53 Multivariable Calculus with Stankova
Section #114; time: 2 – 3:30 pm
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1. Use the Chain Rule to find the indicated partial derivatives.

$$T = \frac{v}{u+2v}, \qquad u = pq\sqrt{r}, \qquad v = p\sqrt{q}r;$$

Find $\partial T/\partial p$, $\partial T/\partial q$, $\partial T/\partial r$ when $p = 1, q = 1, r = 4$.

Solution. We start with the partial derivatives of $T(u, v)$.

$$\begin{aligned} \frac{\partial T}{\partial u} &= \frac{-v}{(u+2v)^2}, \\ \frac{\partial T}{\partial v} &= \frac{(u+2v) - 2v}{(u+2v)^2} \\ &= \frac{-u}{(u+2v)^2}. \end{aligned}$$

At $p = 1$, $q = 1$, $r = 4$, we have $u = 2$, $v = 4$ and

$$\frac{\partial T}{\partial u} = -\frac{1}{25}, \qquad \frac{\partial T}{\partial v} = \frac{1}{50}.$$

Now

$$\begin{aligned} \frac{\partial T}{\partial p} &= \frac{\partial T}{\partial u}\frac{\partial u}{\partial p} + \frac{\partial T}{\partial v}\frac{\partial v}{\partial p} = -\frac{1}{25}(q\sqrt{r}) + \frac{1}{50}(\sqrt{q}r) = 0, \\ \frac{\partial T}{\partial q} &= \frac{\partial T}{\partial u}\frac{\partial u}{\partial q} + \frac{\partial T}{\partial v}\frac{\partial v}{\partial q} = -\frac{1}{25}(p\sqrt{r}) + \frac{1}{50}\left(\frac{pr}{2\sqrt{q}}\right) = -\frac{1}{25}, \\ \frac{\partial T}{\partial r} &= \frac{\partial T}{\partial u}\frac{\partial u}{\partial r} + \frac{\partial T}{\partial v}\frac{\partial v}{\partial r} = -\frac{1}{25}\left(\frac{pq}{2\sqrt{r}}\right) + \frac{1}{50}(p\sqrt{q}) = \frac{1}{100}. \end{aligned}$$

2. *True / False?* There exists a function not differentiable at the origin that is continuous at the origin and has partial derivatives at the origin.

Solution. **True.** The function mentioned in section,

$$f(x, y) = \sqrt[3]{xy}$$

is an example of such function. The continuity follows from f being the composition of a polynomial with a power function. However it is not differentiable at the origin because we cannot find ϵ_1 and $\epsilon_2 \to 0$ such that

$$\sqrt[3]{\Delta x \Delta y} = \epsilon_1 \Delta x + \epsilon_2 \Delta y,$$

which is the definition of differentiability because the partial derivatives at the origin are 0.

Remark. A clearer solution can be obtained through the solutions to the solutions to Quiz 7 for Dis. 117.

3. *True / False?* Suppose $g(x, y)$ is a linear function and $f(x, y)$ is a two-variable function, not necessarily linear. If

$$f(0,0) = g(0,0) \qquad \text{and} \qquad \lim_{(x,y)\to(0,0)} |f(x,y) - g(x,y)| \to 0$$

then g is a good linear approximation to f, so f is a differentiable function.

Solution. **False.** Consider the function $f(x, y) = \sqrt{x^2 + y^2}$ which does not even have partial derivatives. It is not differentiable and thus does not have good linear approximations. However $g(x, y) = 0$ will satisfy the requirements above.