Quiz #8; Tuesday, date: 03/13/2018
MATH 53 Multivariable Calculus with Stankova
Section #114; time: 2 – 3:30 pm
GSI name: Kenneth Hung
Student name: SOLUTIONS

1. Find the absolute maximum and minimum values of f on the set D, where

$$\begin{aligned} f(x, y) &= x^2 y \\ D &= \{(x, y) \mid x \geq 0, y \geq 0, x^2 + y^2 \leq 9\} \end{aligned}$$

Solution. Note that D is a closed and bounded set, so we just need to check all critical points, and find the extremum on the boundary.

We start by taking first derivatives.

$$f_x = 2xy, \qquad f_y = x^2.$$

So any points with $x = 0$ is a critical point. And so the critical points are thus $(0, y)$, where $0 \leq y \leq 3$. At these points, $f(x, y) = 0$.

For the boundary, on $x = 0$ we have $f(x, y) = 0$. On $y = 0$ we have $f(x, y) = 0$. Finally for $x^2 + y^2 = 9$, we have

$$f(x, y) = x^2 y = (9 - y^2)y = 9y - y^3.$$

To find the extremum in this region, we differentiate with respect to y, giving $9 - 3y^2$, which is equal to 0 when $y = \sqrt{3}$. At this point, we have $f(\sqrt{6}, \sqrt{3}) = 6\sqrt{3}$.

Comparing all these values we have found, we conclude the absolute maximum is $6\sqrt{3}$ and the absolute minimum is 0.

2. *True / False?* The normal vector to the surface $z = f(x, y)$ at point $(a, b, f(a, b))$ is

$$\langle f_x(a, b), f_y(a, b), -1 \rangle.$$

Solution. **True.** We can rearrange the equation into $f(x, y) - z = 0$, which is the level surface of the function $F(x, y, z) = f(x, y) - z$. Therefore the normal vector is the gradient of F, which is $\langle f_x(a, b), f_y(a, b), -1 \rangle$.

Alternative solution. **True.** Recall that the tangent plane is given by

$$\begin{aligned} f_x(a, b)(x - a) + f_y(a, b)(y - b) + c &= z \\ f_x(a, b)x + f_y(a, b) - z &= f_x(a, b)a + f_y(a, b)b - c, \end{aligned}$$

giving the normal vector $\langle f_x(a, b), f_y(a, b), -1 \rangle$.

3. *True / False?* Suppose the second partial derivatives of D is continuous on a disk near (a, b). Then for second derivative test, if the determinant $D > 0$ and $f_{yy}(a, b) > 0$, we cannot determine if this is a local minimum or maximum because we do not know the sign of $f_{xx}(a, b)$.

   Solution. **False.** f_{yy} can be used to determine if it is a local maximum or a local minimum as well, in lieu of f_{xx}. In particular, because $D > 0$, we must have
   $$f_{xx} f_{yy} = D + (f_{xy})^2 > 0,$$
   so f_{xx} and f_{yy} must carry the same sign and checking any one of them is sufficient.