Quiz #8; Tuesday, date: 03/13/2018
MATH 53 Multivariable Calculus with Stankova
Section #117; time: 5 – 6:30 pm
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1. Find the directions in which the directional derivative of $f(x, y) = y^3 + yx^2$ at the point $(0, 1)$ has the value 1.

   Solution. The gradient of f is $\langle 2xy, 3y^2 + x^2\rangle$. At $(0, 1)$ this is $\langle 0, 3\rangle$. If a unit vector $\mathbf{u}$ gives a directional derivative of 1, then we must have

   $$\langle 0, 3\rangle \cdot \mathbf{u} = 1.$$

   Suppose $\mathbf{u} = \langle \cos\theta, \sin\theta\rangle$, then

   $$3\sin\theta = 1 \qquad \Longrightarrow \qquad \sin\theta = \frac{1}{3},$$

   which means $\cos\theta = \pm\frac{\sqrt{8}}{3}$. So the unit vector $\mathbf{u}$ is $\langle \pm\frac{\sqrt{8}}{3}, 1\rangle$.

2. *True / False?* The normal vector to the surface $z = f(x, y)$ is three-dimensional, while the normal vector to the level curve of $z = f(x, y)$ is two-dimensional.

   Solution. **True.** The surface $z = f(x, y)$ is a three-dimensional object, so the normal vector lives in the same space. The level curve lives in a two-dimensional space, so the normal vector to that lives in the two-dimensional space as well.

3. *True / False?* The surface $z = x^2 - xy - y^2$ has a saddle point.

   Solution. **True.** We start by finding all critical points. We take first derivatives:

   $$\frac{\partial z}{\partial x} = 2x - y, \qquad \frac{\partial z}{\partial y} = -x - 2y.$$

   Setting both of these to zero gives a unique solution of $(x, y) = (0, 0)$. It remains to check what D is. We have

   $$\frac{\partial^2 z}{\partial x \partial x} = 2, \qquad \frac{\partial^2 z}{\partial x \partial y} = \frac{\partial^2 z}{\partial y \partial x} = -1, \qquad \frac{\partial^2 z}{\partial y \partial y} = -2$$

   so

   $$D = \begin{vmatrix} 2 & -1 \\ -1 & -2 \end{vmatrix} = -5,$$

   which is negative. So $(0, 0)$ is a saddle point.