Quiz #10; Tuesday, date: 04/03/2018
MATH 53 Multivariable Calculus with Stankova
Section #114; time: 2 – 3:30 pm
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1. Use spherical coordinates to evaluate $\iiint_E z^2\,dV$, where E is the solid hemisphere $x^2+y^2+z^2 \le 4$, $y \ge 0$.

   Solution. By Equation 3 on pg. 1047, the integral is

$$\begin{aligned}\iiint_E x^2\,dV &= \int_0^\pi \int_0^\pi \int_0^2 \rho^2\cos^2\phi\rho^2\sin\phi\,d\rho\,d\theta\,d\phi \\ &= \left(\int_0^\pi \cos^2\phi\sin\phi\,d\phi\right)\left(\int_0^\pi d\theta\right)\left(\int_0^2 \rho^4\,d\rho\right) \\ &= \left[-\frac{\cos^3\phi}{3}\right]_0^\pi [\theta]_0^\pi \left[\frac{\rho^5}{5}\right]_0^2 \\ &= \frac{2}{3}\cdot\pi\cdot\frac{32}{5} \\ &= \frac{64}{15}\pi.\end{aligned}$$

2. *True / False?* The volume of the solid enclosed by $z = x^2+y^2-1$ and the plane $z=0$ is given by

$$\int_{-1}^{1}\int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} (1-x^2-y^2)\,dx\,dy$$

   Solution. **True.** The region we will integrate over is the disc $x^2+y^2 \le 1$. Over this region, $z=0$ is in fact on top of $z=x^2+y^2-1$, so the integrand should be $1-x^2-y^2$. The bounds given describes the region.

3. *True / False?* For a region R, the integral $\iint_R dA$ gives the area of R.

   Solution. **True.** This is analogous to integrating $\iiint_R dV$ giving the volume in triple integrals.