Quiz #10; Tuesday, date: 04/03/2018
MATH 53 Multivariable Calculus with Stankova
Section #117; time: 5 – 6:30 pm
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1. Find the volume of the solid enclosed by $z = x^2 + y^2 - 1$ and the planes $x = 0$, $y = 0$, $z = 0$ and $x + y = 1$.

   Solution. Note that over this region, $z = 0$ is on top of $z = x^2 + y^2 - 1$, so the volume is given by the integral

$$\begin{aligned}\int_0^1 \int_0^{1-x} (1 - x^2 - y^2)\, dy\, dx &= \int_0^1 \left[y - x^2 y - \frac{y^3}{3} \right]_0^{1-x} dx \\ &= \int_0^1 \left((1-x) - x^2(1-x) - \frac{(1-x)^3}{3} \right) dx \\ &= \left[-\frac{(1-x)^2}{2} - \frac{x^3}{3} + \frac{x^4}{4} + \frac{(1-x)^4}{12} \right]_0^1 \\ &= \frac{1}{3}.\end{aligned}$$

2. *True / False?* The integral

$$\iint_R f(x, y)\, dA$$

   over the triangular region bounded by the x-, y- axes and the line $x+y = 1$ cannot be rewritten as a double integral using polar coordinates.

   Solution. **False.** It is not easy but it can be rewritten using polar coordinates. The limits will look like

$$\int_0^{\pi/2} \int_0^{\sec(\theta - \pi/4)/\sqrt{2}} \cdots\, dr\, d\theta.$$

3. *True / False?* The transformation from Cartesian coordinates to cylindrical coordinates is given by

$$x = r\cos\theta, \quad y = r\sin\theta, \quad z = h.$$

   The Jacobian determinant is r.

   Solution. **True.** The Jacobian can be computed directly as

$$\begin{vmatrix} \cos\theta & -r\sin\theta & 0 \\ \sin\theta & r\cos\theta & 0 \\ 0 & 0 & 1 \end{vmatrix} = r.$$