**Quiz #11; Tuesday, date: 04/10/2018**
**MATH 53 Multivariable Calculus with Stankova**
**Section #114; time: 2 – 3:30 pm**
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**Student name: SOLUTIONS**

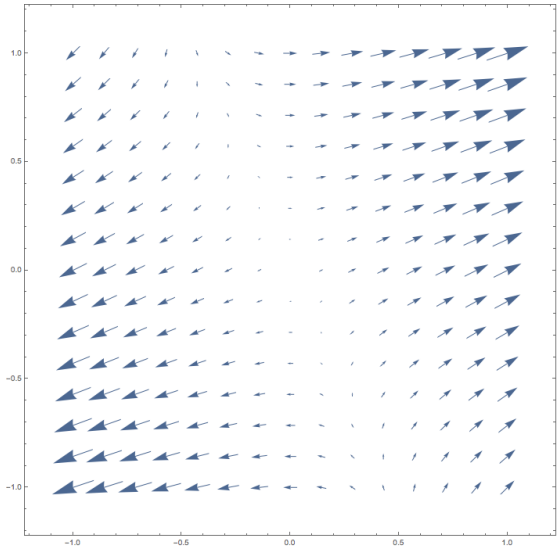
1. Find the gradient vector field $\nabla f$ of $f$ and sketch it.

$$f(x, y) = x(x + y)$$

*Solution.* We take the gradient directly.

$$\nabla f(x, y) = \langle 2x + y, x \rangle$$

To sketch it, we start by checking the vector field on the axis. On the $x$-axis, the vector is $\langle 2x, x \rangle$. So they are all parallel but with different magnitude / direction. On the $y$-axis, the vector is $y, 0$. Inspired by this, we can also observe that along the line $2x + y = 0$, the gradient vector is always vertical. Putting all these information in the sketch gives




2. *True / False?* $\mathbf{F}(x, y) = \langle x^2, y^2 \rangle$ is a conservative vector field.

*Solution.* **True.** It can be written as the gradient of the function $\frac{1}{3}x^3 + \frac{1}{3}y^3$.

3. *True / False?* Suppose $f$ is a scalar function and $\nabla f$ is a force field. The work done by this force field along any one level curve of $f$ maybe nonzero.

*Solution.* **False.** The gradient is always perpendicular to the level curves. So the tangent along any level curve is perpendicular to the force field at all times. Since $\mathbf{F} \cdot \mathbf{T} = 0$, there is no work done.