Quiz #11; Tuesday, date: 04/10/2018
MATH 53 Multivariable Calculus with Stankova
Section #117; time: 5 – 6:30 pm
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1. Evaluate the line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$, where C is given by the vector function $\mathbf{r}(t)$.

   $$\mathbf{F}(x, y, z) = -y\mathbf{i} + (x+y)\mathbf{j} - \frac{1}{2}(x^2+y^2)\mathbf{k}, \quad \mathbf{r} = \langle \cos t, \sin t, t \rangle, \quad 0 \leq t \leq 2\pi$$

   Solution. We compute the integral directly.

   $$\begin{aligned} \int_0^{2\pi} \langle -\sin t, \cos t + \sin t, -\frac{1}{2} \rangle \cdot \langle -\sin t, \cos t, 1 \rangle \, dt &= \int_0^{2\pi} \left(\sin t \cos t + \frac{1}{2} \right) dt \\ &= \left[\frac{\sin^2 t}{2} + \frac{t}{2} \right]_0^{2\pi} \\ &= \pi. \end{aligned}$$

2. *True / False?* If we overlay a sketch of the gradient vector field ∇f and the contour map f, the arrows from the vector field will always be perpendicular to the contour lines.

   Solution. **True.** The contour lines are the level curves and the arrows are the gradients. Recall that the normal vector to the level curves are precisely the gradients, so these two objects are perpendicular to each other.

3. *True / False?* Suppose f is a nonnegative function and C is the curve parametrized as

   $$x = a + (b - a)t, \quad y = 0, \quad 0 \leq t \leq 1$$

   Then $\int_C f(x, y)\, ds \geq 0$ but $\int_a^b f(x, y)\, dx$ maybe negative.

   Solution. **True.** The first integral is always positive, as it is

   $$\int_0^1 f(x, y)\sqrt{(b-a)^2 + 0^2}\, dt = \int_0^1 f(x, y)|b - a|\, dt,$$

   whose integrand is never negative. On the other hand, the second integral might be negative if $b < a$.