Quiz #12; Tuesday, date: 04/17/2018
MATH 53 Multivariable Calculus with Stankova
Section #117; time: 5 – 6:30 pm
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1. Show that the line integral is independent of path and evaluate the integral.
$$\int_C -\cos y\,dx + (x\sin y - \cos y)\,dy,$$
where C is any path from $(3,0)$ to $(1,\pi)$.

   Solution. The vector field $\mathbf{F} = \langle \cos y, x\sin y - \cos y\rangle$ is conservative because x- and y- component has continuous first-order partial derivatives everywhere, the domain is open and simply-connected, and
$$\frac{\partial}{\partial x}(x\sin y - \cos y) = \sin y = \frac{\partial}{\partial y}(-\cos y).$$
We proceed to find the function f such that $\nabla f = \mathbf{F}$. Observe that
$$\nabla(-x\cos y - \sin y) = \langle \cos y, x\sin y - \cos y\rangle,$$
so $f(x,y) = -x\cos y - \sin y$ is the potential function. Now by Fundamental Theorem of Line Integral we have
$$\int_C -\cos y\,dx + (x\sin y - \cos y)\,dy = f(1,\pi) - f(3,0) = 1 - (-3) = 4.$$

2. *True / False?* Fix two points A and B in a domain D. If $\int_C \mathbf{F}\cdot d\mathbf{r}$ is the same for all paths C from A to B, then $\mathbf{F}$ must be conservative on D.

   Solution. **False.** The domain D may be not be connected. A and B maybe not be connected which makes the given condition vacuously true. Even A and B are connected, this will not help us for the part of the domain that are not connected to either of these.

3. *True / False?* Here is another proof of Green's Theorem with holes in it: Suppose the region with hole is D', the hole itself is D_2 and the region D' with the hole filled is D_1. The outer and inner boundaries are C_1 and C_2. We can then apply Green's Theorem to D_1 and D_2, and subtract one integral from the other.

   Solution. **False.** This would require Green's Theorem to hold for the hole, which may not be true, especially if P and Q do not have continuous first-order partial derivative for some points in the hole.