Quiz #13; Tuesday, date: 04/24/2018
MATH 53 Multivariable Calculus with Stankova
Section #114; time: 2 – 3:30 pm
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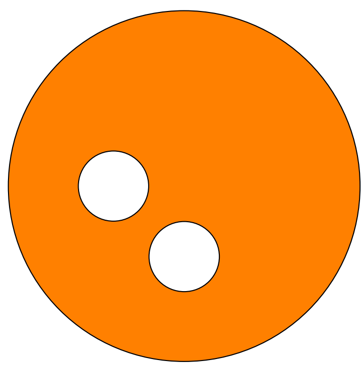
1. Is there a vector field $\mathbf{G}$ on $\mathbb{R}^3$ such that $\operatorname{curl}\mathbf{G} = \langle \cos x, \sin y, z\rangle$? Explain.

   Solutions. We start by taking the divergence of this vector field. Since the divergence of the curve of nice functions is always 0, this is sufficient to show that the vector field is not the curl of any $\mathbf{G}$. Now

$$\operatorname{div}\langle \cos x, \sin y, z\rangle = -\sin x + \cos y + 1,$$

   which is not constant zero. Hence the vector field cannot be written as $\operatorname{curl}\mathbf{G}$.

2. *True / False?* Given a vector field $\mathbf{F} = \langle P, Q\rangle$ with $\frac{\partial Q}{\partial x} = \frac{\partial P}{\partial y}$ over the region given below.



   Suppose the big circle is C_1 and the small circles are C_2 and C_3, all counterclockwise, then

$$\int_{C_1} \mathbf{F}\cdot d\mathbf{r} = \int_{C_2} \mathbf{F}\cdot d\mathbf{r} + \int_{C_3} \mathbf{F}\cdot d\mathbf{r}.$$

   Solution. **True.** We can cut this into three pieces of simply connected region that allows us to apply Green's theorem.

3. *True / False?* Surface of revolution of a positive differentiable function is always smooth.

   Solution. **True.** The surface of revolution can be parametrized by

$$x = x, \quad y = f(x)\cos\theta, \quad z = f(x)\sin\theta, \quad 0 \le \theta \le 2\pi.$$

The tangent vectors are

$$\begin{aligned} \mathbf{r}_x &= \langle 1, f'(x)\cos\theta, f'(x)\sin\theta \rangle, \\ \mathbf{r}_\theta &= \langle 0, -f(x)\sin\theta, f(x)\cos\theta \rangle. \end{aligned}$$

If we take the cross product, this gives

$$\mathbf{r}_x \times \mathbf{r}_\theta = \langle f(x)f'(x), -f(x)\cos\theta, -f(x)\sin\theta \rangle,$$

which is never $\mathbf{0}$.