Worksheet #2; date: 01/23/2018
MATH 53 Multivariable Calculus

1. *(Stewart 10.1.33)* Find parametric equations for the path of a particle that moves along the circle $x^2 + (y-1)^2 = 4$ in the manner described.
   (a) Once around clockwise, starting at $(2, 1)$
   (b) Three times around counterclockwise, start at $(2, 1)$
   (c) Halfway around counterclockwise, starting at $(0, 3)$
2. *(Stewart 10.3.5)* Consider the points $(-4, 4)$ and $(3, 3\sqrt{3})$, given in Cartesian coordinates.
   (a) Find the polar coordinate (r, θ) of the point, where $r > 0$ and $0 \leq \theta < 2\pi$.
   (b) Find the polar coordinate (r, θ) of the point, where $r < 0$ and $0 \leq \theta < 2\pi$
3. *(Stewart 10.3.12)* Sketch the region in the plane consisting of points whose polar coordinates satisfy the given conditions.
$$r \geq 1, \quad \pi \leq \theta \leq 2\pi$$
4. *(Stewart 10.3.23)* Find a polar equation for the curve represented by the given Cartesian equation.
$$y = 1 + 3x.$$
5. *(Stewart 10.3.27; modified)* Find both the Cartesian equation and the polar equation for each of the following described curves. Decide for yourself which one would you rather do / use.
   (a) A line through the origin that makes an angle of $\pi/6$ with the positive axis.
   (b) A vertical line through the point $(3, 3)$.
6. *(Challenging; Stewart 10.3.52; modified)* Rewrite the curve $(x^2 + y^2)^2 = 4x^2y^2$ in polar coordinates. Sketch the curve afterwards.
7. *(Stewart 10.2.3)* Find an equation of the tangent to the curve at the point corresponding to the given value of the parameter.
$$x = t^3 + 1, \quad y = t^4 + t; \quad t = -1.$$
8. *(Stewart 10.2.7)* Find an equation of the tangent to the curve at the given point by two methods: (a) without eliminating the parameter and (b) by first eliminating the parameter.
$$x = 1 + \ln t, \quad y = t^2 + 2; \quad (1, 3).$$

9. *(Stewart 10.2.16)* Find dy/dx and d^2y/dx^2. For which values of t is the curve concave upward?

$$x = \cos t, \qquad y = \sin 2t, \qquad 0 < t < \pi.$$

10. *(Stewart 10.2.25)* Show that the curve $x = \cos t$, $y = \sin t \cos t$ has two tangents at $(0, 0)$ and find their equations. Sketch the curve.