Worksheet #4; date: 01/23/2018
MATH 53 Multivariable Calculus

1. *(Stewart 12.1.15)* Find an equation of the sphere that passes through the point $(4, 3, -1)$ and has center $(3, 8, 1)$.

2. *(Stewart 12.1.31)* Describe in words the region of $\mathbb{R}^3$ represented by the equations
$$x^2 + y^2 = 4, \quad z = -1.$$

3. *(Stewart 12.1.42)* Write inequalities to describe the region: the solid upper hemisphere of the sphere of radius 2 centered at the origin.

4. *(Stewart 12.2.21)* Find $\mathbf{a} + \mathbf{b}$, $4\mathbf{a} + 2\mathbf{b}$, $|\mathbf{a}|$, and $|\mathbf{a} - \mathbf{b}|$, where
$$\mathbf{a} = 4\mathbf{i} - 3\mathbf{j} + 2\mathbf{k}, \quad \mathbf{b} = 2\mathbf{i} - 4\mathbf{k}.$$

5. *(Stewart 12.2.41)* Find the unit vectors that are parallel to the tangent line to the parabola $y = x^2$ at the point $(2, 4)$.

6. Draw any quadrilateral $ABCD$. Use vectors to prove that the line joining the midpoints of AB and the midpoints of BC is parallel to the line joining the midpoints of DA and the midpoints of CD.
(Challenging) show that the midpoints of the four sides form a parallelogram.

7. *(Stewart 12.3.9)* Find $\mathbf{a} \cdot \mathbf{b}$ where
$$|\mathbf{a}| = 7, \quad |\mathbf{b}| = 4, \quad \text{the angle between } \mathbf{a} \text{ and } \mathbf{b} \text{ is } 30^\circ.$$

8. *(Stewart 12.3.19)* Find the dot product, and thus the angle between the vectors
$$\mathbf{a} = 4\mathbf{i} - 3\mathbf{j} + \mathbf{k}, \quad \mathbf{b} = 2\mathbf{i} - \mathbf{k}.$$

9. *(Stewart 12.3.27)* Find a unit vector that is orthogonal to both $\mathbf{i} + \mathbf{j}$ and $\mathbf{i} + \mathbf{k}$.