Worksheet #5; date: 02/01/2018
MATH 53 Multivariable Calculus

1. *(Stewart 12.3.37; modified)* Find the direction cosines and direction angles of the vector $\langle c, c, c\rangle$, where $c > 0$. How would your answer change if $c < 0$? Verify that $\cos^2\alpha + \cos^2\beta + \cos^2\gamma = 1$.

2. *(Stewart 12.3.41)* Find the scalar and vector projections of $\mathbf{b}$ onto $\mathbf{a}$, where
$$\mathbf{a} = \langle 4, 7, -4\rangle, \quad \mathbf{b} = \langle 3, -1, 1\rangle.$$

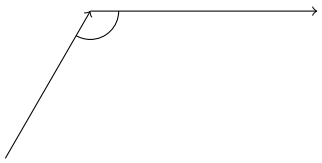
3. Find the angle between a diagonal of a cube and an edge incidental to it.

4. *(Stewart 12.3.63)* The Parallelogram Law states that
$$|\mathbf{a} + \mathbf{b}|^2 + |\mathbf{a} - \mathbf{b}|^2 = 2|\mathbf{a}|^2 + 2|\mathbf{b}|^2.$$
   (a) Give a geometric interpretation of the Parallelogram Law.
   (b) Prove the Parallelogram Law.

5. *(Stewart 12.4.5)* Find the cross product of
$$\mathbf{a} = \frac{1}{2}\mathbf{i} + \frac{1}{3}\mathbf{j} + \frac{1}{4}\mathbf{k}, \quad \mathbf{b} = \mathbf{i} + \mathbf{2j} - \mathbf{3k}.$$

6. *(Stewart 12.4.7)* Find the cross product of
$$\mathbf{a} = \langle t, 1, 1/t\rangle, \quad \mathbf{b} = \langle t^2, t^2, 1\rangle.$$

7. *(Stewart 12.4.15)* Find $|\mathbf{u} \times \mathbf{v}|$ and determine whether $\mathbf{u} \times \mathbf{v}$ is directed into the page or out of the page.



   The length of the horizontal arrow is 16 and the length of the other arrow is 12. The angle marked is 120°.

8. *(From earlier worksheet)* Find a unit vector orthogonal to both $\mathbf{i} + \mathbf{j}$ and $\mathbf{i} + \mathbf{k}$.

9. Use the scalar triple product to show that the vectors
$$\mathbf{u} = \mathbf{i} + 5\mathbf{j} - 2\mathbf{k}, \quad \mathbf{v} = 3\mathbf{i} - \mathbf{j}, \quad \mathbf{w} = 5\mathbf{i} + 9\mathbf{j} - 4\mathbf{k}$$
   are coplanar.

10. *(Stewart 12.4.43)* If $\mathbf{a}\cdot\mathbf{b} = \sqrt{3}$ and $\mathbf{a}\times\mathbf{b} = \langle 1, 2, 2\rangle$, find the angle between $\mathbf{a}$ and $\mathbf{b}$.