

Worksheet #16; date: 03/13/2018
MATH 53 Multivariable Calculus

1. (*Stewart 14.7.33*) Find the absolute maximum and minimum values of f on the set D .

$$f(x, y) = x^2 + y^2 + x^2y + 4,$$

$$\text{and } D = \{(x, y) \mid |x| \leq 1, |y| \leq 1\}.$$

2. (*Stewart 14.7.45*) Find three positive numbers whose sum is 100 and whose product is a maximum.
3. (*Stewart 14.7.53*) A cardboard box without a lid is to have a volume of 32000 cm^3 . Find the dimensions that minimize the amount of cardboard used.
4. Quiz time!
5. (*Stewart 14.8.29*) Use Lagrange multipliers to prove that the rectangle with maximum area that has a given parameter p is a square.
6. Solve Stewart 14.7.53 (given above) by Lagrange multiplier.
7. (*Challenging; modified; Stewart 14.7.57*) The Shannon index (sometimes called the Shannon-Wiener index or Shannon-Weaver index) is a measure of diversity in an ecosystem. For the case of three species, it is defined as

$$H = -p_1 \ln p_1 - p_2 \ln p_2 - p_3 \ln p_3$$

where p_i is the proportion of species i in the ecosystem. Using the fact that $p_1 + p_2 + p_3 = 1$, and considering the domain of H , find the maximum value of H . For what values of p_1, p_2, p_3 does it occur?