Worksheet #17; date: 03/15/2018
MATH 53 Multivariable Calculus

1. *(Stewart 14.8.44)* Find the maximum and minimum volumes of a rectanglar box whole surface area is 1500 cm^2 and whose total edge length is 200 cm.

2. *(Stewart 14.8.45)* The plane $x + y + 2z = 2$ intersects the paraboloid $z = x^2 + y^2$ in an ellipse. Find the points on this ellipse that are nearest to and farther from the origin.

3. *(Teaser for 14.8.49)*

   (a) Find the minimum value of

   $$f(x_1, x_2, x_3, x_4, x_5) = \sqrt{\frac{x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2}{5}}$$

   given that x_1, x_2, x_3, x_4, x_5 are positive numbers and $x_1 + x_2 + x_3 + x_4 + x_5 = c$, where c is a constant.

   (b) Deduce from part (a) that if x_1, x_2, x_3, x_4, x_5 are positive numbers, then

   $$\sqrt{\frac{x_1^2 + x_2^2 + x_3^2 + x_4^2 + x_5^2}{5}} \geq \frac{x_1 + x_2 + x_3 + x_4 + x_5}{5}.$$

   This inequality says that the root-mean-square of 5 numbers is no larger than the arithmetic mean of the numbers. Under what circumstances are these two means equal?

4. Evaluate the double integral by first identifying it as the volume of a solid.

   $$\iint_R (3 - y)\, dA, \qquad R = [0, 1] \times [0, 1].$$

5. The integral $\iint_R \cos y\, dA$, where $R = [0, 3] \times [0, \pi/2]$, represents the volume of a solid. Sketch the solid and find its volume.