

Worksheet #21; date: 04/05/2018
MATH 53 Multivariable Calculus

1. (*Concept check*) If \mathbf{F} is a force field, what is its input? What is its output?
2. *True / False?* The volume of the solid enclosed by $z = x^2 + y^2 - 1$ and the plane $z = 0$ is given by

$$\int_{-1}^1 \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} (1 - x^2 - y^2) dx dy$$

3. *True / False?* For a region R , the integral $\iint_R dA$ gives the area of R .
4. *True / False?* The integral

$$\iint_R f(x, y) dA$$

over the triangular region bounded by the x -, y - axes and the line $x+y = 1$ cannot be rewritten as a double integral using polar coordinates.

5. *True / False?* The transformation from Cartesian coordinates to cylindrical coordinates is given by

$$x = r \cos \theta, \quad y = r \sin \theta, \quad z = h.$$

The Jacobian determinant is r .

6. (*Stewart 16.1.9*) Sketch the vector field \mathbf{F} by drawing a diagram.

$$\mathbf{F}(x, y, z) = -y\mathbf{i}$$

7. (*Stewart 16.1.21*) Find the gradient vector field of f and sketch it.

$$f(x, y) = y \sin(xy)$$

8. (*Stewart 16.1.25*) Find the gradient vector field of f and sketch it.

$$f(x, y) = \frac{1}{2}(x^2 - y^2)$$

9. (*Stewart 16.1.33*) A particle moves in a velocity field $\mathbf{V}(x, y) = \langle x^2, x+y^2 \rangle$. If it is at position $(2, 1)$ at time $t = 3$, estimate its location at time $t = 3.01$.