Worksheet #23; date: 04/12/2018
MATH 53 Multivariable Calculus

1. *True / False?* $\mathbf{F}(x, y) = \langle x^2, y^2 \rangle$ is a conservative vector field.

2. *True / False?* Suppose f is a scalar function and ∇f is a force field. The work done by this force field along any one level curve of f is zero.

3. *True / False?* If we overlay a sketch of the gradient vector field ∇f and the contour map f, the arrows from the vector field will always be perpendicular to the contour lines.

4. *True / False?* Suppose f is a nonnegative function and C is the curve parametrized as

$$x = a + (b - a)t, \quad y = 0, \quad 0 \leq t \leq 1$$

Then $\int_C f(x, y)\, ds \geq 0$ but $\int_a^b f(x, y)\, dx$ maybe negative.

5. *(Stewart 16.3.7)* Determine whether or not $\mathbf{F}$ is a conservative vector field. If it is, find a function f such that $\mathbf{F} = \nabla f$.

$$\mathbf{F}(x, y) = (ye^x + \sin y)\mathbf{i} + (e^x + x \cos y)\mathbf{j}$$

6. *(Stewart 16.3.9)* Determine whether or not $\mathbf{F}$ is a conservative vector field. If it is, find a function f such that $\mathbf{F} = \nabla f$.

$$(y^2 \cos x + \cos y)\mathbf{i} + (2y \sin x - x \sin y)\mathbf{j}$$

7. *(Stewart 16.3.17)* Find a function f such that $\mathbf{F} = \nabla f$ and use this to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ along the given curve $\mathbf{C}$.

$$\mathbf{F}(x, y, z) = yze^{xz}\mathbf{i} + e^{xz}\mathbf{j} + xye^{xz}\mathbf{k},$$

where C is $\mathbf{r}(t) = (t^2 + 1)\mathbf{i} + (t^2 - 1)\mathbf{j} + (t^2 - 2t)\mathbf{k}$, $0 \leq t \leq 2$.

8. *(Stewart 16.3.23)* Find the work done by the force field $\mathbf{F}$ in moving an object from P to Q.

$$\mathbf{F}(x, y) = x^3\mathbf{i} + y^3\mathbf{j}, \quad P(1, 0), Q(2, 2)$$

9. *(Stewart 16.3.31)* Determine whether or not the given set is open, is connected, and is simply-connected.

$$\{(x, y) \mid 0 < y < 3\}$$

10. *(Stewart 16.3.33)* Determine whether or not the given set is open, is connected, and is simply-connected.

$$\{(x, y) \mid 1 \leq x^2 + y^2 \leq 4, y \geq 0\}$$