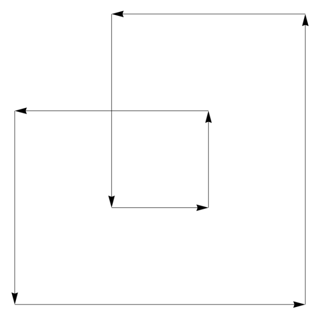
Worksheet #25; date: 04/19/2018
MATH 53 Multivariable Calculus

1. *True / False?* Fix two points A and B in a simply connected domain D. If $\int_C \mathbf{F} \cdot d\mathbf{r}$ is the same for all paths C from A to B, then $\mathbf{F}$ must be conservative on D.

2. *True / False?* Suppose P and Q has continuous partial derivatives everywhere. Green's Theorem cannot help us in computing line integral $\int_C \mathbf{F} \cdot d\mathbf{r}$ where C is given below.



3. *True / False?* Fix two points A and B in a domain D. If $\int_C \mathbf{F} \cdot d\mathbf{r}$ is the same for all paths C from A to B, then $\mathbf{F}$ must be conservative on D.

4. *True / False?* Here is another proof of Green's Theorem with holes in it: Suppose the region with hole is D', the hole itself is D_2 and the region D' with the hole filled is D_1. The outer and inner boundaries are C_1 and C_2. We can then apply Green's Theorem to D_1 and D_2, and subtract one integral from the other.

5. *(Concept check)* What are the symbols for grad, div, curl respectively? What sort of objects does the operator ∇, $\nabla\cdot$, $\nabla\times$ and ∇^2 take as input? What sort of objects are the output? Write ∇^2 in terms of the other operators.

6. *(Stewart 16.5.7)* Compute the curl and the divergence of the vector field

$$\mathbf{F}(x, y, z) = \langle e^x \sin y, e^y \sin z, e^z \sin x\rangle$$

7. *(Stewart 16.5.17)* Determine whether or not the vector field is conservative. If it is conservative, find a function f such that $\mathbf{F} = \nabla f$.

$$\mathbf{F}(x, y, z) = e^{yz}\mathbf{i} + xze^{yz}\mathbf{j} + xye^{yz}\mathbf{k}$$

8. *(Stewart 16.5.19)* Is there a vector field $\mathbf{G}$ on $\mathbf{R}^3$ such that curl$\mathbf{G}$ = $\langle x \sin y, \cos y, z - xy\rangle$? Explain.

9. *(Concept check)* What is “conservative”, “irrotational” and “incompressible”?

10. *(Stewart 16.5.27)* Prove the identity

$$\text{div}(\mathbf{F} \times \mathbf{G}) = \mathbf{G} \cdot \text{curl } \mathbf{F} - \mathbf{F} \cdot \text{curl } \mathbf{G}$$

11. *(Stewart 16.5.29)* Prove the identity

$$\text{curl}(\text{curl } \mathbf{F}) = \text{grad}(\text{div } \mathbf{F}) - \nabla^2 \mathbf{F}$$