**Worksheet #27; date: 04/26/2018**
**MATH 53 Multivariable Calculus**

1. *(Stewart 16.7.9)* Evaluate the surface integral
$$\iint_S x^2yz\,dS,$$
where $S$ is the part of the plane $2x + 2y + z = 4$ that lies above the rectangle $[0, 3] \times [0, 2]$.

2. *(Stewart 16.7.17)* Evaluate the surface integral
$$\iint_S (x^2z + y^2z)\,dS,$$
where $S$ is the hemisphere $x^2 + y^2 + z^2 = 4$, $z \geq 0$.

3. *(Stewart 16.7.25)* Evaluate the surface integral (a.k.a. flux) $\iint_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F} = x\mathbf{i} + y\mathbf{j} + z^2\mathbf{k}$, and $S$ is the sphere with radius 1 and center the origin, oriented outwards.

4. *(Stewart 16.7.27; setup only)* Evaluate the surface integral (a.k.a. flux) $\iint_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F} = y\mathbf{j} - z\mathbf{k}$, and $S$ consists of the paraboloid $y = x^2 + z^2$, $0 \leq y \leq 1$, and the disk $x^2 + y^2 \leq 1$, $y = 1$, oriented outwards.

5. What are the conditions for Stokes'?

6. Similarity between Stokes' and Green's: $\operatorname{curl} \mathbf{F} \approx Q_x - P_y$. For Green's, think about $\langle P, Q, 0\rangle$. This similarity shows up in verifying conservativeness too!

7. *(Stewart 16.8.5)* Use Stokes' Theorem to evaluate $\iint_S \operatorname{curl} \mathbf{F} \cdot d\mathbf{S}$, where $\mathbf{F} = xyz\mathbf{i} + xy\mathbf{j} + x^2yz\mathbf{k}$, and $S$ consists of the top and the four sides (but not the bottom) of the cube with vertices $(\pm 1, \pm 1, \pm 1)$, oriented outwards.

8. *(Stewart 16.8.9)* Use Stokes' Theorem to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F}(x, y, z) = xy\mathbf{i} + yz\mathbf{j} + zx\mathbf{k}$, and $C$ is the boundary of the part of the paraboloid $z = 1 - x^2 - y^2$ in the first octant, oriented counterclockwise as viewed from above.