

Worksheet #28; date: 05/01/2018
MATH 53 Multivariable Calculus

1. (Stewart 16.Rev.38) Let

$$\mathbf{F}(x, y) = \frac{(2x^3 + 2xy^2 - 2y)\mathbf{i} + (2y^3 + 2x^2y + 2x)\mathbf{j}}{x^2 + y^2}.$$

Evaluate $\oint_C \mathbf{F} \cdot d\mathbf{r}$, where C is a curve looped around the origin once counterclockwise.

2. (Stewart 16.7.43) A fluid has density 870 kg/m^3 and flows with velocity $\mathbf{v} = z\mathbf{i} + y^2\mathbf{j} + x^2\mathbf{k}$, where x , y , and z are measured in meters and the components of \mathbf{v} in meters per second. Find the rate of flow outward through the cylinder $x^2 + y^2 = 4$, $0 \leq z \leq 1$.
3. *True / False?* If a vector field $\vec{F}(x, y, z)$ is conservative on an open region in \mathbb{R}^3 that contains an oriented, smooth surface S with a simple, closed, smooth boundary curve C , then even if we orient C negatively, both sides of ST will yield 0.
4. *True / False?* When we change the variables in a double integral using a transformation $T : x = g(u, v), y = h(u, v)$ that sends the original domain $D \subset \mathbb{R}_{x,y}^2$ to $S \subset \mathbb{R}_{u,v}^2$, we need to ensure that T is one-to-one but not necessarily onto.
5. *True / False?* When trying to calculate/find the limit $\lim_{(x,y) \rightarrow (a,b)} f(x, y) = L$, we insist on $(x, y) \neq (a, b)$ because the limit may exist at (a, b) even if the function is not defined or discontinuous there.
6. *True / False?* By changing the x, y -coordinate system, we can see that the graph of function $f(x, y) = xy$ is actually a parabolic hyperboloid.