Worksheet #1; date: 08/26/2018
MATH 55 Discrete Mathematics

1. *True / False?* Propositions can be thought of as a variable which may be true or false.

2. *True / False?* In propositional logic, a logical operator is a function that takes two input proposition and outputs a truth value.

3. *(Rosen 1.1.13a–f)* Let p and q be the propositions

   $$p : \text{You drive over 65 per hour.}$$
   $$q : \text{You get a speeding ticket.}$$

   Write these propositions using p and q and logical connectives (including negations).

   (a) You do not drive over 65 miles per hour.

   (b) You drive over 65 miles per hour, but you do not get a speeding ticket.

   (c) You will get a speeding ticket if you drive over 65 miles per hour.

   (d) If you do not drive over 65 miles per hour, then you will not get a speeding ticket.

   (e) Driving over 65 miles per hour is sufficient for getting a speeding ticket.

4. Consider this sentence: I come back on campus only if I have teaching or meeting.

   (a) Rephrase it in the form "if p then q"

   (b) What is the converse, contrapositive and inverse of your answer to (a).

5. Solve the system of equations:

   $$\begin{cases} x^2 - 4x + 3 & \neq & 0 \\ x^3 - 5x^2 + 6x & \geq & 0 \end{cases}$$

6. Without using a truth table, explain why the following proposition is always false.

   $$(p \oplus q) \wedge (q \oplus r) \wedge (r \oplus p)$$

7. *(Rosen 1.3.10c, 12c)* Show that

   $$[p \wedge (p \to q)] \to q$$

   is tautology

(a) with a truth table;

(b) without a truth table.

(c) *(Extra)* What does this proposition being tautology mean?

8. *(Rosen 1.3.25)* Show that $(p \to r) \vee (q \to r)$ and $(p \wedge q) \to r$ are logically equivalent. How does this help us in writing proofs?

9. *(Rosen 1.3.33)* Show that $(p \to q) \to (r \to s)$ and $(p \to r) \to (q \to s)$ are not logically equivalent.