Worksheet #2; date: 08/29/2018
MATH 55 Discrete Mathematics

1. *True / False?* The negation of "all heroes wear cape" is "there exists a hero who does not wear cape".

2. *True / False?* All trees that can walk will also eat you.

3. *(Rosen 1.4.36c)* Find a counterexample, if possible to these universally quantified statements, where the domain for all variables consists of all integers.
$$\forall x(|x| < 0)$$

4. What do we need to do to show the following statement?

   There exists a prime number that is larger that 10^6.

5. *(Rosen 1.4.43)* Determine whether $\forall x(P(x) \to Q(x))$ and $\forall x\, P(x) \to \forall x\, Q(x)$ are logically equivalent. Justify your answer.

6. *(Rosen 1.5.23d)* Express the following mathematical statements using predicates, quantifiers, logical connectives, and mathematical operators.

   A negative real number does not have a square root that is a real number.

7. *(Rosen 1.5.49a)* Show that $\forall x\, P(x) \wedge \exists x\, Q(x)$ equivalent to $\forall x \exists y(P(x) \wedge Q(y))$, where all quantifiers have the same nonempty domain.

8. *(Rosen 1.6.10e)* What relevant conclusions can be drawn from the following set of premise?

   All foords that are healthy to eat do not taste good.

   Tofu is healthy to eat.

   You only eat what tastes good.

   You do not eat tofu.

   Cheeseburgers are not healthy to eat.

9. *(Rosen 1.6.19)* Determine whether each of these arguments is valid. If it is not correct, what logical error occurs?

   (a) If n is a real number such that $n > 1$, then $n^2 > 1$. Suppose that $n^2 > 1$. Then $n > 1$.

   (b) If n is a real number with $n > 3$, then $n^2 > 9$. Suppose that $n^2 \leq 9$. Then $n \leq 3$.

   (c) If n is a real number with $n > 2$, then $n^2 > 4$. Suppose that $n \leq 2$. Then $n^2 \leq 4$.