

**Worksheet #3; date: 09/05/2018**  
**MATH 55 Discrete Mathematics**

1. (*Rosen 1.7.7*) Use a direct proof to show that every odd integer is the difference of two squares.
2. (*Rosen 1.7.8*) Prove that if  $n$  is a perfect square, then  $n+2$  is not a perfect square.
3. (*Rosen 1.7.13*) Prove that if  $x$  is irrational, then  $1/x$  is irrational.
4. (*Rosen 1.7.39*) Prove that at least one of the real numbers  $a_1, a_2, \dots, a_n$  is greater than or equal to the average of these numbers. What kind of proof did you use?
5. (*Rosen 1.8.17*) Suppose that  $a$  and  $b$  are odd integers with  $a \neq b$ . Show there is a unique integer  $c$  such that  $|a - c| = |b - c|$ .
6. (*Rosen 1.8.25*) Write the numbers  $1, 2, \dots, 2n$  on a blackboard, where  $n$  is an odd integer. Pick any two of the numbers,  $j$  and  $k$ , write  $|j - k|$  on the board and erase  $j$  and  $k$ . Continue this process until only one integer is written on the board. Prove that this integer must be odd.
7. (*Rosen 1.8.34*) Prove that  $\sqrt[3]{2}$  is irrational.
8. (*Rosen 1.8.43*) Prove that you can use dominoes to tile a rectangular checkerboard with an even number of squares.