Worksheet #4; date: 09/10/2018
MATH 55 Discrete Mathematics

1. *(Remark)* You are going to see $\subseteq$ mixed with $\subset$. Do not be surprised in the future if you see $\subset$ to mean subset and $\subsetneq$ for proper subset. Notation is a mess.

2. What is the cardinality of $\mathcal{P}(\{a, b, c\})$?

3. (a) *(Rosen 2.1.38)* Show that $A \times B \neq B \times A$, when A and B are nonempty, unless $A = B$.
   (b) Show that $A \times B \subseteq B \times A$ if and only if $A = B$.

4. *(Set theory definition of integer)* Suppose $S_0 = \emptyset$, and for every positive integer i, we define $S_i = S_{i-1} \cup \{S_{i-1}\}$.
   (a) Write down S_1, S_2 and S_3.
   (b) Is $S_1 \subseteq S_3$? If so, is $S_1 \subset S_3$?
   (c) *(Challenging)* Show that $j \leq k$ is a necessary and sufficient condition for $S_j \subseteq S_k$.

5. *(Rosen 2.2.18c)* Let A, B, and C be sets. Show that
$$(A - B) - C \subseteq A - C.$$

6. *(Rosen 2.2.51)* Find $\bigcup_{i=1}^{\infty} A_i$ and $\bigcap_{i=1}^{\infty} A_i$ if for every positive integer i,
   (a) $A_i = \{-i, -i+1, \ldots, -1, 0, 1, \ldots, i-1, i\}$.
   (b) $A_i = \{-i, i\}$.
   (c) $A_i = [-i, i]$, that is, the set of real numbers x with $-i \leq x \leq i$.
   (d) $A_i = [i, \infty)$, that is, the set of real numbers x with $x \geq i$.
   (e) *(Extra; challenging)* $A_i = (-1/i, 1/i)$.

7. What can we say about f and g if the graph of f is a proper subset of the graph of g?

8. *(Rosen 2.3.15a–b)* Determine whether $f : \mathbb{Z} \times \mathbb{Z} \to \mathbb{Z}$ is onto if
   (a) $f(m, n) = m + n$.
   (b) $f(m, n) = m^2 + n^2$.

9. *(Rosen 2.3.74a–d)* Prove or disprove each of these statements about the floor and ceiling functions.
   (a) $\lfloor \lceil x \rceil \rfloor = \lceil x \rceil$ for all real numbers x.
   (b) $\lfloor x + y \rfloor = \lfloor x \rfloor + \lfloor y \rfloor$.
   (c) $\lceil \lceil x/2 \rceil / 2 \rceil = \lceil x/4 \rceil$ for all real numbers x.