Worksheet #5; date: 09/12/2018
MATH 55 Discrete Mathematics

1. *(Rosen 2.3.34) True / False?* If f and $f \circ g$ are one-to-one, then g must be one-to-one.

2. *(Challenging; Rosen 2.3.80)* Show that a set S is infinite if and only if there is a proper subset A of S such that there is a one-to-one correspondence between A and S.

3. *(Rosen 2.4.35, 36)* Use the identity

$$\frac{1}{k(k+1)} = \frac{1}{k} - \frac{1}{k+1}$$

to compute $\sum_{k=1}^{n} \frac{1}{k(k+1)}$.

4. *(Rosen 2.4.41)* Find a formula for $\sum_{k=0}^{m} \lfloor \sqrt{k} \rfloor$.

5. *(Rosen 2.5.3)* Deterine whether each of these sets is countable or uncountable. For those that are countably infinite, exhibit a one-to-one-correspondence between the set of positive integers and that set.

   (a) all bit strings not containing the bit 0

   (b) all positive rational numbers that cannot be written with denominators less than 4

   (c) the real numbers not containing 0 in their decimal representation

   (d) the real numbers containing only a finite number of 1s in their decimal representation

6. *(Rosen 2.5.11)* Give an example of two uncountable sets A and B such that $A \cap B$ is

   (a) finite.

   (b) countably infinite.

   (c) uncountable.

7. *(Rosen 2.5.17)* If A is an uncountable set and B is a countable set, must $A - B$ be uncountable?

8. *True / False?* $|\mathbb{R}| = |\mathbb{C}|$