

**Worksheet #6; date: 09/17/2018**  
**MATH 55 Discrete Mathematics**

1. (*Rosen 4.1.7*) Show that if  $a$ ,  $b$ , and  $c$  are integers, where  $a \neq 0$  and  $c \neq 0$ , such that  $ac \mid bc$ , then  $a \mid b$ .
2. (*Rosen 4.1.37a*) Find counterexample to the follow statement about congruences: If  $ac \equiv bc \pmod{m}$ , where  $a$ ,  $b$ ,  $c$ , and  $m$  are integers with  $m \geq 2$ , then  $a \equiv b \pmod{m}$ .
3. (*Rosen 4.1.38*) Show that if  $n$  is an integer then  $n^2 \equiv 0$  or  $1 \pmod{4}$ .
4. (*Rosen 4.1.39*) Use the question above to show that if  $m$  is a positive integer of the form  $4k + 3$  for some nonnegative integer  $k$ , then  $m$  is not the sum of the squares of two integers.
5. (*Rosen 4.2.23c*) Find the sum and product of the following pairs of numbers. Express your answers as an octal expansion.

$$(1111)_8, (777)_8$$

6. Show that a positive integer is divisible by 9 if and only if the sum of its decimal digits is divisible by 9.
7. (*Challenging*) Suppose we want to measure an item that weighs  $n$  pounds, where  $n$  is some unknown nonnegative integer no greater than 1000. We are given a balance that has two sides to it, and we get to choose the mass of the weights used. Due to a budget crisis, we want to minimize the number of weights ordered. How do we achieve this, and how many weights are needed?