Worksheet #7; date: 09/19/2018
MATH 55 Discrete Mathematics

1. *(Rosen 4.3.11)* Show that $\log_2 3$ is an irrational number. Recall that an irrational number is a real number x that cannot be written as the ratio of two integers.

2. *(Rosen 4.3.17c)* Determine whether the integers in the following set are pairwise relatively prime:
$$12, 17, 31, 37$$

3. *(Rosen 4.3.25a, f)* What are the greatest common divisors of these pairs of integers?

   (a) $3^7 \cdot 5^3 \cdot 7^3$, $2^{11} \cdot 3^5 \cdot 5^9$

   (f) 1111, 0

4. *(Rosen 4.3.33e, f; modified)* Use Euclidean algorithm to find

   (e) $\gcd(1000, 5040)$

   (f) $\gcd(9888, 6060)$

   Write the gcd as a linear combination of the two numbers as well.

5. *(Rosen 4.3.50)* Show that if a, b and m are integers such that $m \geq 2$ and $a \equiv b \pmod{m}$, then $\gcd(a, m) = \gcd(b, m)$.

6. *(Rosen 4.3.55)* Adapt the proof in the text that there are infinitely many primes to prove that there are infinitely many primes of the form $4k + 3$, where k is a nonnegative integer. *Hint:* Suppose that there are only finitely many such primes $q_1, q_2, \ldots, q_n$, and consider the number $4q_1q_2 \cdots q_n - 1$.