Worksheet #8; date: 09/25/2018
MATH 55 Discrete Mathematics

1. *(Rosen 4.4.6c)* Find an inverse of a modulo m for the following pairs of relatively prime integers.

$$a = 144, \quad m = 233.$$

2. *(Rosen 4.4.7)* Show that if a and m are relatively prime positive integers, then the inverse of a modulo m is unique modulo m. *Hint:* Assume that there are two solutions b and c of the congruence $ax \equiv 1 \pmod{m}$, and show that $b \equiv c \pmod{m}$.

3. *(Rosen 4.4.12b)* Solve the following congruences using the modular inverses found in Question 1:

$$144x \equiv 4 \pmod{233}$$

4. *(Rosen 4.4.21, 24)* Find all solutions to the system of congruences $x \equiv 1 \pmod{2}$, $x \equiv 2 \pmod{3}$, $x \equiv 3 \pmod{5}$, and $x \equiv 4 \pmod{11}$ with two methods:

   (a) The construction in the proof of the Chinese remainder theorem

   (b) Back substitution.

5. *(Challeging)* This questions put all techniques above together. Try this now, or after the lecture!

$$\begin{cases} 9x + 1 \equiv 0 \pmod{2} \\ 5x \equiv 1 \pmod{3} \\ x \equiv 4 \pmod{7} \end{cases}$$