Worksheet #11; date: 10/03/2018
MATH 55 Discrete Mathematics

1. *(Rosen 5.1.14)* Prove that for every positive integer n,

$$\sum_{k=1}^{n} k2^k = (n-1)2^{n+1} + 2.$$

2. *(Rosen 5.1.21)* Prove that $2^n > n^2$ if n is an integer greater than 4.

3. *(Rosen 5.1.51)* What is wrong with this "proof"?

   "Theorem". For every positive integer n, if x and y are positive integers with $\max(x, y) = n$, then $x = y$.

   Basis step. Suppose that $n = 1$. If $\max(x, y) = 1$ and x and y are positive integers, we have $x = 1$ and $y = 1$.

   Inductive step. Let k be a positive integer. Assume that whenever $\max(x, y) = k$ and x and y are positive integers, then $x = y$. Now let $\max(x, y) = k+1$, where x and y are positive integers. Then $\max(x-1, y-1) = k$, so by the inductive hypothesis, $x - 1 = y - 1$. It follows that $x = y$, completing the inductive step.

4. *(Rosen 5.1.72; simplified)* Show that it is possible to arrange the numbers $1, 2, \ldots, 2^k$ in a row so that the average of any two of these numbers never appears between them. What if the numbers are $1, 2, \ldots, n$ instead?