

Worksheet #12; date: 10/08/2018
MATH 55 Discrete Mathematics

1. (*Rosen 5.2.7*) Which amounts of money can be formed using just two-dollar bills and five-dollar bills? Prove your answer using strong induction.
2. (*Rosen 5.2.17*) Use strong induction to show that if a simple polygon with at least four sides is triangulated, then at least two of the triangles in the triangulation have two sides that border the exterior of the polygon.
3. (*Rosen 5.2.18; challenging*) Use strong induction to show that when a simple polygon P with consecutive vertices v_1, v_2, \dots, v_n is triangulated into $n-2$ triangles, the triangles can be numbered $1, 2, \dots, n-2$ such that v_i is a vertex of triangle i for $i = 1, 2, \dots, n-2$.
4. (*Rosen 5.2.29*) What is wrong with this “proof” by strong induction?
“*Theorem*”. For every nonnegative integer n , $5n = 0$.
Basis step. $5 \cdot 0 = 0$.
Inductive step. Suppose $5j = 0$ for all nonnegative integers j with $0 \leq j \leq k$. Write $k+1 = i+j$, where i and j are natural numbers less than $k+1$. By inductive hypothesis, $5(k+1) = 5(i+j) = 5i + 5j = 0 + 0 = 0$.
5. (*Rosen 5.3.6b, e*) Determine whether each of these proposed definitions is a valid recursive definition of a function f from the set of nonnegative integers to the set of integers. If f is well defined, find a formula for $f(n)$ when n is a nonnegative integer and prove that your formula is valid.
 - (a) $f(0) = 1, f(1) = 0, f(2) = 2, f(n) = 2f(n-3)$ for $n \geq 3$
 - (b) $f(0) = 2, f(n) = f(n-1)$ if n is odd and $n \geq 1$ and $f(n) = 2f(n-2)$ if $n \geq 2$