

**Worksheet #20; date: 11/05/2018**  
**MATH 55 Discrete Mathematics**

1. (*Planarity game*) <https://www.jasondavies.com/planarity/>
2. (*Rosen 10.7.13*) Suppose that a connected planar graph has six vertices, each of degree four. Into how many regions is the plane divided by a planar representation of this graph?
3. (*Rosen 10.7.17*) Suppose that a connected planar simple graph with  $e$  edges and  $v$  vertices contains no simple circuits of length 4 or less. Show that  $e \leq (5/3)v - (10/3)$  if  $v \geq 4$ .
4. What is the chromatic number of  $K_{m,n}$  and  $Q_n$ ?
5. Give an example of a graph where the highest degree is larger than the chromatic number.
6. An *edge coloring* of a graph is an assignment of colors to the edges so that edges incident with a common vertex are assigned different colors. Show that the highest degree is at most the edge chromatic number.
7. Find the edge chromatic numbers of  $C_n$  and  $K_n$ .
8. The *crossing number* of a simple graph is the minimum number of crossings that can occur when this graph is drawn in a plane where no three arcs representing edges are permitted to cross at the same point. If graph  $G$  has a crossing number of 100, what is its minimum possible chromatic number?