

# Qualifying exam syllabus

Kenneth Hung

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## COMMITTEE

David Aldous, Steve Evans, William Fithian (Stat) , James Demmel (Chair)

## MAJOR TOPIC: PROBABILITY THEORY (PROBABILITY)

**Preliminaries**  $\sigma$ -algebras, Dynkin's  $\pi$ - $\lambda$  theorem, Independence, Borel–Cantelli lemma, Kolmogorov's 0-1 law, Kolmogorov's maximal inequality, Strong and weak laws of large numbers

**Central limit theorems** Weak convergence, Uniform integrability, Characteristic functions, I.I.D. central limit theorem, Lindeberg–Feller CLT

**Conditioning** Conditional probability, Conditional expectation, Conditional independence, Regular conditional probabilities

**Martinagles** Stopping times, Upcrossing inequality, A.S. convergence, Doob's decomposition, Doob's inequality,  $L^p$  convergence,  $L^1$  convergence, Reverse martingale convergence, Optional stopping theorem, Wald's identity

**Markov chains** Countable state space, Stationary measures, Convergence theorems, Recurrence and transience, Asymptotic behavior

**References** Durrett, Probability: Theory and Examples, Chapters 1–3, 5–6

## MAJOR TOPIC: THEORETICAL STATISTICS (PROBABILITY)

**Exponential family** Densities, Parameters, Moments, Cumulants, Generating functions

**Estimators and statistics** Risks, Estimators, Sufficient statistic, Complete statistic, Ancillary statistic, Factorization theorem, Minimal sufficient statistic, Basu's theorem, Rao–Blackwell theorem

**Unbiased estimation** UMVU estimators, Variance bounds, Fisher information, Cramér–Rao bound, Higher dimension variance bounds

**Bayesian estimation** Prior and posterior distribution, conjugate distribution

**Large sample theory** Convergence in probability, Convergence in distribution, Central limit theorem, Delta method, Asymptotic relative efficiency

**Estimating equations** Weak law for random functions, Kullback–Leibler divergence, Consistency of maximum likelihood estimator, limiting distribution for MLE, Confidence interval, asymptotic confidence interval

**Empirical Bayes** Empirical Bayes estimator, James–Stein estimator

**Hypothesis testing** Test function, Power, Significance, Neyman–Pearson lemma, Uniformly most powerful tests, Monotone likelihood ratio, Duality between testing and interval estimation, Generalized Neyman–Pearson lemma, Two-sided hypothesis, Unbiased test

**References** Keener, Theoretical Statistics, Chapters 2–4, 6–9, 11

MINOR TOPIC: NUMERICAL LINEAR ALGEBRA (APPLIED MATHEMATICS)

**Linear equation solving** Gaussian elimination, Perturbation theory, Blocking algorithms, Special linear systems (real symmetric positive definite matrices, symmetric indefinite matrices, general sparse matrices)

**Linear least squares** Normal equations, QR decomposition, Singular value decomposition, Householder transformations, Givens rotations, Rank-deficient least square problems, Perturbation theory

**Nonsymmetric eigenproblem** Power method, Inverse iteration, Orthogonal iteration, QR iteration, Tridiagonal and bidiagonal reduction, Perturbation theory

**Symmetric eigenproblem** Tridiagonal QR iteration, Rayleigh quotient iteration, Divide-and-conquer, Bisection and inverse iteration, Perturbation theory

**References** Demmel, Applied Linear Algebra, Chapters 2–5